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A Three-Dimensional Axisymmetric Calculation Procedure for Turbulent Flows in a Radial Vaneless Diffuser

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A THREE-DIMENSIONAL AXISYMMETRIC CALCULATION PROCEDURE FOR TURBULENT FLOWS IN A RADIAL VANELESS DIFFUSER

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E-2387

ABSTRACT

An analytical model is proposed to calculate the three-dimensional axisymmetric turbulent flowfield in a radial vaneless diffuser. The model assumes that the radial and tangential boundary layer profiles can be approximated by power law profiles. Then, using the integrated radial and tangential momentum and continuity equations for the boundary layer and corresponding inviscid equations for the core flow, there results six ordinary differential equation in six unknowns which can easily be solved using a Runge-Kutta technique. A model is also proposed for fully developed flow. The results using this technique have been compared with the results from a three-dimensional viscous, axisymmetric duct code and with experimental data and good quantitative agreement was obtained.

NOMENCLATURE

a	speed of sound
A	area
c_f	skin friction coefficient
c_p	specific heat ratio at constant pressure
c_{pr}	pressure coefficient, $\frac{P - P_2}{P'_{2e} - P_2}$
F	entrainment function
h	diffuser half-width
H_1	shape factor, $\frac{\delta - \delta_1}{\delta_2}$
H_2	shape factor, $\frac{\delta_1}{\delta_2}$
m	exponent in radial velocity profile
$\dot{m}$	mass flow
M	Mach number
n	exponent in tangential velocity profile
p	pressure
p_{STD}	standard pressure, 101325 N/m
r	radius
R	dimensionless radial distance, $\frac{r - r_2}{r_3 - r_2}$
R_g	gas constant
Re	Reynold's number
s	entropy
s	streamline distance
T	temperature
T_{STD}	standard temperature, 288.2 K
v	velocity
y	coordinate normal to the wall
Y	dimensionless y-distance, y/h
α	flow angle from radial direction
γ	specific heat ratio
δ	boundary layer thickness
δ_{1r}	displacement thickness, $\int_0^\delta \left(1 - \frac{v_r}{v_{re}}\right) dy$

δ_{1s} displacement thickness, $\int_0^\delta \left(1 - \frac{v}{v_e}\right) dy$

δ_{2r} momentum thickness, $\int_0^\delta \frac{v_r}{v_{re}}\left(1 - \frac{v_r}{v_{re}}\right) dy$

δ_{2s} momentum thickness, $\int_0^\delta \frac{v}{v_e}\left(1 - \frac{v}{v_e}\right) dy$

μ coefficient of viscosity

ρ density

τ shear stress

ϕ entropy function, $\int c_p \frac{dT}{T}$

Subscripts:

c centerline

cr critical

e edge of boundary layer

r radial direction

R reference condition

s streamwise direction

w wall

y direction normal to the wall

θ tangential direction

1 compressor inlet

2 vaneless diffuser inlet

3 vaneless diffuser exit

Superscripts:

' total conditions

($\bar{\ }$) mass-averaged conditions

INTRODUCTION

The flowfield in the radial vaneless diffuser of a centrifugal compressor is extremely complex since the flow is turbulent, unsteady, viscous, and three-dimensional. Also, depending on the initial state of the end-wall boundary layers and the diffuser length, the flow may become fully developed (i.e., the end-wall boundary layers may merge in the center of the channel) or may separate off one of the walls. One of the earliest attempts to calculate this flowfield was due to Stanitz (1). Other investigators have used a similar approach to calculate the frictional losses in the vaneless diffuser (2,3). In Stanitz' model, the flow is assumed to be one-dimensional with the losses accounted for by using a shear force term proportional to an assigned friction factor in the radial and tangential momentum equations. This approach has several weaknesses. First of all, the development of the end-wall boundary layers is not accounted for except through the inclusion of an effective channel height term. This generally leads to an overprediction of the static pressure rise unless an unrealistically high friction factor is used. Secondly, the method gives no information about the velocity profiles and flow angle distribution at the exit of the vaneless diffuser. This information would be helpful in the design of the leading edge of the diffuser vanes. Finally, the choice of an appropriate friction factor and the effective passage height distribution are totally arbitrary and will often vary from one machine to another.

More recently, several investigators (4-6) have attempted to calculate the three-dimensional boundary layers in vaneless diffusers. The major drawback of these methods is the empiricism used in the specification of the velocity profiles. This affects the calculation of the boundary layer shape factor and therefore also the skin friction and the location of possible separation points.

Currently, there are computer codes (7) which can calculate the three-dimensional flow fields in annular ducts. These codes can also be used for calculating the flow field in a radial vaneless diffuser but their running times and complexity preclude their use as design tools. Also, in the case of the ADD code (7), there are stringent Mach number and Reynold's number limitations.

In the present paper, a three-dimensional axisymmetric calculation procedure for turbulent flows in a radial vaneless diffuser is presented. The method is a noniterative integral method and is therefore much faster than finite difference methods. The primary assumption made is that both the axial and tangential boundary layer velocity profiles can be described by power laws; however, the exponents in these power laws are allowed to vary with radius as prescribed by the equations of motion. Compressibility is accounted for in the radial direction but not in the direction normal to the end-walls. Turbulent flow is assumed everywhere using the semi-empirical Ludwieg-Tillman relation for skin-friction. A method for calculating fully-developed flows is also presented. Comparisons are made to results from the ADD code for both the boundary layer analysis and for the fully developed flow analysis. Comparisons are also made with experimental data in the literature. In all cases, good agreement is demonstrated.

ANALYSIS

The type of vaneless diffuser geometry considered by this method of analysis is shown in Fig. 1. Typical velocity profiles for the cases of boundary layer flow and fully developed flow are shown in Figs. 2(a) and (b).

Boundary Layer Flow

The following assumptions are made for this analysis:

(1) The flow is steady and turbulent everywhere.
(2) The fluid is a perfect gas.
(3) The flow is axisymmetric.
(4) The end-walls of the vaneless diffuser are adiabatic.
(5) There is no density variation in the direction normal to the vaneless diffuser end-walls although the density is allowed to vary in the radial direction, i.e., $\rho = \rho(r)$.
(6) The vaneless diffuser geometry and flow are symmetric about the diffuser centerline.

(7) The boundary layer velocity profiles in the radial and tangential directions may be described by the power law equations

$$\frac{v_r}{v_{re}} = \left(\frac{y}{\delta}\right)^m ; \frac{v_\theta}{v_{\theta e}} = \left(\frac{y}{\delta}\right)^n \tag{1}$$

where v_{re}, $v_{\theta e}$, δ, m, and n are functions of radius only.

The boundary layer equations of motion in cylindrical coordinates for axisymmetric turbulent flow are as follows:

Continuity

$$\frac{\partial(\rho v_r)}{\partial r} + \frac{\rho v_r}{r} + \frac{\partial(\rho v_y)}{\partial y} = 0 \tag{2}$$

Radial Momentum

$$v_r \frac{\partial v_r}{\partial r} - \frac{v_\theta^2}{r} + v_y \frac{\partial v_r}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{1}{\rho}\frac{\partial \tau_r}{\partial y} \tag{3}$$

Tangential Momentum

$$v_r \frac{\partial v_\theta}{\partial r} + \frac{v_r v_\theta}{r} + v_y \frac{\partial v_\theta}{\partial y} = \frac{1}{\rho}\frac{\partial \tau_\theta}{\partial y} \tag{4}$$

y-Momentum

$$\frac{\partial p}{\partial y} = 0 \tag{5}$$

If the power law profiles of Eq. (1) are assumed, the problem reduces to that of determining the free stream velocity components v_{re} and $v_{\theta e}$; the boundary layer thickness, δ; the boundary layer profile exponents, m and n; and the free stream thermodynamic properties, p, ρ, and T_e. Thus eight equations are required to completely solve the flow field. Three of these equations are obtained by integrating Eqs. (2) to (4) through the boundary layer which results in the following

$$\int_0^\delta \frac{\partial(\rho v_r)}{\partial r} dy + \frac{1}{r}\int_0^\delta \rho v_r dy + \int_0^\delta \frac{\partial(\rho v_y)}{\partial y} dy = 0 \tag{6}$$

$$\int_0^\delta \frac{\partial(\rho v_r v_y)}{\partial y} + \int_0^\delta \frac{\partial(\rho v_r^2)}{\partial r} dy - \frac{1}{r}\int_0^\delta \rho(v_\theta^2 - v_r^2) dy = -\delta\frac{\partial p}{\partial r} - \tau_{rw} \tag{7}$$

$$\int_0^\delta \frac{\partial(\rho v_r v_\theta)}{\partial r} dy + \int_0^\delta \frac{\partial(\rho v_y v_\theta)}{\partial y} dy + \int_0^\delta \frac{2\rho v_r v_\theta}{r} dy = -\tau_{\theta w} \tag{8}$$

Equations (3,4) can also be evaluated for the isentropic core flow. This results in the following two equations

$$\frac{dp}{dr} = -\rho v_{re}\frac{dv_{re}}{dr} + \rho\frac{v_{\theta e}^2}{r} \tag{9}$$

$$\frac{dv_{\theta e}}{dr} = -\frac{v_{\theta e}}{r} \tag{10}$$

From the boundary layer assumption, which is inherent in Eq. (5), the static pressure is constant across the channel and thus is a function of radius only.

The remaining three equations are obtained from channel continuity, the energy equation, and the equation of state. Channel continuity can be written

$$\dot{m} = 2\rho v_{re}(2\pi r)(h - \delta_{1r}) \tag{11}$$

where for a power law profile

$$\delta_{1r} = \frac{m}{1+m}\delta$$

Differentiating Eq. (11) with respect to the radius yields an expression for dv_{re}/dr. If the total temperature is assumed to be constant in the vaneless diffuser, which is true if there are no inlet normal gradients of total temperature, the energy equation for the core flow can be written

$$T_e = T' - \frac{v_e^2}{2c_p} \tag{12}$$

The final equation which completes the system is the equation of state for the core flow which is

$$p = \rho R_g T_e \tag{13}$$

One additional equation is needed to solve this system since an additional unknown is added in the integration of Eqs. (2) to (4); i.e., the normal velocity at the edge of the boundary layer, v_{ye}. This can be modelled using Head's entrainment relation which empirically predicts the rate at which fluid from the free stream is entrained into the boundary layer. Head showed that the entrainment rate can be modelled as a function only of a boundary layer shape factor. Sumner and Shanebrook (8) extended the definition to three-dimensional compressible flow as follows

$$F = \frac{1}{v_e}\left(v_e\frac{d\delta}{ds} - v_{ye}\right) = 0.0306\ (H_1 - 3)^{-0.653} \tag{14}$$

Finally, expressions for the wall shearing stresses can be given by

$$\tau_{rw} = \frac{\rho v_{re}^2}{\cos\alpha_e}\frac{c_f}{2} \tag{15}$$

$$\tau_{\theta w} = \frac{\rho v_{re}^2}{\cos\alpha_e}\tan\alpha_e\frac{c_f}{2} \tag{16}$$

where c_f is given by the Ludwieg-Tillman relation which is

$$c_f = 0.246\ R_{e_{\delta 2r}}^{-0.268}\left(\frac{T}{T^*}\right)_e^{-1.561\ H_2} \tag{17}$$

Using the velocity profiles from Eq. (1) and performing the indicated differentiations and integrations, Eqs. (6) to (8) and (11) can be combined to yield the following equations

$$\left[\left(1 - M_{re}^2\right) + 2m(1 + 2m)\frac{\delta}{h}\right]\frac{1}{v_{re}}\frac{dv_{re}}{dr}$$

$$= -\frac{M_{\theta e}^2 + 1}{r} - \frac{1}{h}\frac{dh}{dr}$$

$$+ 2m(1 + 2m)\frac{\delta}{h}\left[\frac{n(1 + 2m)}{m(1 + 2n)}\frac{\tan^2\alpha_e}{r} + \frac{c_f(1 + 2m)}{4m\delta\cos\alpha_e}\right]$$

$$- 2m^2\frac{F}{h\cos\alpha_e} \tag{18}$$

$$\frac{dm}{dr} = m(1 + m)(1 + 2m)\left[-\frac{2}{v_{re}}\frac{dv_{re}}{dr} + \frac{2n(1 + 2m)}{m(1 + 2n)}\frac{\tan^2\alpha_e}{r}\right.$$

$$\left. + \frac{c_f(1 + 2m)}{2m\delta\cos\alpha_e} - \frac{F}{\cos\alpha_e}\right] \tag{19}$$

$$\frac{d\delta}{dr} = -\delta\left[(1 - M_{re}^2)\frac{1}{v_{re}}\frac{dv_{re}}{dr} + \frac{M_{\theta e}^2 + 1}{r}\right]$$

$$+ \frac{\delta}{1 + m}\frac{dm}{dr} + \frac{(1 + m)F}{\cos\alpha_e} \tag{20}$$

$$\frac{dn}{dr} = \frac{n}{1 + m}\frac{dm}{dr} + \frac{(1 + m + n)^2}{\delta\cos\alpha_e}\left[\frac{c_f}{2} - \frac{nF}{(1 + m + n)}\right] \tag{21}$$

Equations (9), (10), and (18) to (21) can then be solved sequentially for the six unknowns (v_{re}, m, δ, n, p, and $v_{\theta e}$) using a fourth order Runge-Kutta technique such as that described in Ref. (9). Equation (18) is good only for symmetric vaneless diffusers with moderate wall curvatures and uniform core flows. However, Eqs. (9), (10), and (19) to (21) are good for nonuniform core conditions and could be solved together with the equations describing the core flow as an interactive boundary layer problem. The above system of equations is also good for incompressible flow if the Mach numbers, M_{re} and $M_{\theta e}$, are set equal to zero in Eqs. (18) and (20).

A complete derivation of the final form of the continuity equation is described in the Appendix. The radial and tangential momentum equations are derived similarly.

Fully Developed Flow

The system of equations described in the previous section for boundary layer flow breaks down when the boundary layers on the two end-walls meet in the center of the channel, i.e., when $\delta \geq h$. In this case, the flow is said to be fully developed and the velocity profiles are assumed to be as shown in Fig. 2(b).

The following assumptions are made for fully developed flow in addition to those already made for boundary layer flow. (1) The boundary layer equations, Eqs. (2) to (5), remain valid for fully developed flow. (2) The velocity profiles can be described by the power law equations

$$\frac{v_r}{v_{rc}} = \left(\frac{y}{h}\right)^m \; ; \; \frac{v_\theta}{v_{\theta c}} = \left(\frac{y}{h}\right)^n \tag{22}$$

where v_{rc}, $v_{\theta c}$, h, m, and n are functions of radius only. (3) Analagous to fully developed, turbulent, pipe flow theory (10), the shear stress is assumed to vary linearly across the channel.

For fully developed flow, an isentropic core flow no longer exists. Therefore Eqs. (9) and (10) are no longer valid. However, with the assumption of a linear variation of shear stress across the channel, Eqs. (3) and (4) can be evaluated along the channel centerline which yields the equations

$$\frac{dp}{dr} = -\rho v_{rc}\frac{dv_{rc}}{dr} + \frac{\rho v_{\theta c}^2}{r} - \frac{\tau_{rw}}{h} \tag{23}$$

$$\frac{dv_{\theta c}}{dr} = -\frac{v_{\theta c}}{r} - \frac{1}{\rho v_{rc}}\frac{\tau_{\theta w}}{h} \tag{24}$$

where τ_{rw} and $\tau_{\theta w}$ are given by Eqs. (15) and (16) and c_f is given by Eq. (17) with

$$\delta_{2r} = \frac{H_2 - 1}{H_2(H_2 + 1)}h$$

If the continuity equation is integrated across the vaneless diffuser half-width, the following equation results

$$\frac{d}{dr}\int_0^h \rho v_r dy = 0 \tag{25}$$

For the power law profiles of Eq. (22), this becomes

$$\frac{1}{r} + \frac{1}{\rho}\frac{d\rho}{dr} + \frac{1}{v_{rc}}\frac{dv_{rc}}{dr} + \frac{1}{h}\frac{dh}{dr} - \frac{1}{1 + m}\frac{dm}{dr} = 0 \tag{26}$$

The radial momentum equation, Eq. (3), can also be integrated across the diffuser half-width. This becomes, after using the results from Eqs. (23) and (26) and the velocity profiles from Eq. (22)

$$\frac{dm}{dr} = -2m(1 + m)(1 + 2m)\left[\frac{1}{v_{rc}}\frac{dv_{rc}}{dr} - \frac{n(1 + 2m)}{m(1 + 2n)}\frac{\tan^2\alpha_c}{r}\right] \tag{27}$$

When the tangential momentum equation is integrated across the diffuser half-width, the following equation results

$$\frac{dn}{dr} = \frac{n}{1 + m}\frac{dm}{dr} + \frac{(m + n)(1 + m + n)}{\cos\alpha_c}\frac{c_f}{2h} \tag{28}$$

The density variation in the throughflow direction can be obtained from a combination of the equation of state and the energy equation. Then, using the above relations, the following equation for the variation of the centerline radial velocity component results.

$$\left[\left(1 - M_{rc}^2\right) + 2m(1 + 2m)\right] \frac{1}{v_{rc}} \frac{dv_{rc}}{dr} = \frac{M_{\theta c}^2 + 1}{r} - \frac{1}{h}\frac{dh}{dr}$$

$$+ \frac{2n(1 + 2m)^2}{1 + 2n} \frac{\tan^2\alpha_c}{r} + \left[(\gamma - 1)M_{\theta c}^2 + \gamma M_{rc}^2\right] \frac{c_f}{2h \cos \alpha_c} \tag{29}$$

Equation (29) together with Eqs. (23), (24), (27), and (28) then forms a system of five ordinary differential equations in five unknowns (v_{rc}, $v_{\theta c}$, m, n, and p) which can be solved using a fourth-order Runge-Kutta technique.

Separated Flow

By examining Eqs. (19) and (27), it can be seen that the radial velocity profile exponent, m, and therefore also the shape factor, H_2, will increase rapidly if there is a large rate of deceleration or if the flow angle becomes large. This usually means that flow separation is imminent. A large deceleration rate is caused by a rapid increase in flow area. A large flow angle can be the result of the impeller design but will more often be the result of a low mass flow rate through the compressor. In the latter case, a large flow separation may occur in the vaneless space which could precipitate compressor stall and subsequent surge of the system.

The methods outlined in the previous two sections for boundary layer flow, and fully developed flow, could be used to calculate through a separated region of the flow. However, if this were done, large values of the velocity profile exponent, m, would be calculated which would result in unrealistic velocity profiles. A possible way of treating this problem would be to assume a constant value for the exponent, m, after separation. Separation could be determined by examining the shape factor, H_2, or the skin friction coefficient, c_f. The problem could then be treated in the inverse mode using either Eq. (19) for boundary layer flow or Eq. (27) for fully developed flow to determine the variation of free-stream radial velocity. However, since power law profiles cannot describe negative velocities which occur when the flow separates, it would probably be necessary to use some other formulation for the separated flow problem.

Initial Conditions

In order to solve Eqs. (9), (10), and (18) to (21) for boundary layer flow, initial values for the six variables (p, δ, v_{re}, $v_{\theta e}$, m, and n) must be specified. The value of the static pressure can be obtained from experimental data or from an analysis of the impeller flowfield. In the latter case, the pressure could be obtained from the equation

$$p = p_1 \, e^{\frac{\phi - \phi_1 - \Delta s}{R_g}} \tag{30}$$

The entropy rise, Δs, in Eq. (30) should include the entropy rise due to the mixing of the impeller blade surface, boundary layers, wakes, and any other losses which are assumed to affect the impeller core flow.

The values of δ and m can only be obtained from an analysis of the impeller end-wall boundary layers. Ordinarily the value of m should be on the order of 0.2 or 0.3 if there is no great deceleration of the meridional velocity component in the impeller. However, δ will probably be a significant portion of the impeller exit half-width.

If the compressor mass flow is specified, the core flow radial velocity component can then be calculated from Eq. (11). However, the core flow tangential velocity component and corresponding velocity profile exponent are more difficult to determine. A boundary layer analysis of the impeller would give values for the relative tangential velocity component and its corresponding exponent. However, as the fluid moves from the relative frame of reference of the impeller (assuming a shrouded impeller) to the absolute frame of the vaneless diffuser, a large shear occurs on the fluid as it attempts to decelerate from a velocity near the wheel speed to zero near the end-walls. Thus it is impossible to model the velocity profile in this region by a power law. Experimental data does show, however, that a velocity profile that can be approximated by a power law is quickly established. Experimental data also shows (4) that an exponent of n 0.1 adequately describes the tangential boundary layer at the vaneless diffuser inlet. The core flow tangential velocity component can then be obtained from a conservation of tangential momentum analysis. In any event, the initial values of the exponents, m and n, do not have a large effect on the final solution.

For the case of fully developed flow entering the vaneless diffuser, the same variables must be specified as for the boundary layer case with the exception of the boundary layer thickness (which is equal to the diffuser half-width, h). For this case, the value of Δs in Eq. (30) should also include the additional core flow loss due to a nonzero shear stress gradient at the centerline. The values of the other variables are then calculated in the same way as for boundary layer flow.

Loss Calculation

The loss in a vaneless diffuser can be expressed most easily and most rigorously by an increase in the mass-averaged entropy of the fluid. The mass-averaged entropy at a specified radius in the vaneless diffuser can be expressed

$$\overline{\left(s - s_{2e}\right)} = \frac{2\rho \int_0^{\delta} (s - s_{2e})\, v_r (2\pi r)\, dy}{\dot{m}} \tag{31}$$

where the local entropy is given by

$$s - s_{2e} = c_p \ln \frac{\dfrac{T}{T_{2e}}}{\left(\dfrac{p}{p_2}\right)^{\frac{\gamma - 1}{\gamma}}} \tag{32}$$

The entropy rise through the diffuser can then be expressed

$$\overline{\left(s_3 - s_2\right)} = \overline{\left(s - s_{2e}\right)}_3 - \overline{\left(s - s_{2e}\right)}_2 \tag{33}$$

where the values of $(s - s_{2e})_2$ and $(s - s_{2e})_3$ are obtained by a numerical integration of Eq. (32) across the boundary layer. Since the total temperature in the vaneless diffuser is assumed to be constant everywhere, the mass-averaged total pressure ratio can be expressed ordinary differential equations. The continuity equation will serve as an example of how this is done. Also, the continuity equation shows how the entrainment function is used in the derivation. If Leibnitz' rule is used to interchange differentiation and integration, Eq. (6) becomes

$$\frac{1}{\rho v_{re}} \frac{d}{dr} \int_0^\delta \rho v_r dy + \frac{1}{r} \int_0^\delta \frac{v_r}{v_{re}} dy = \frac{1}{v_{re}} \left(v_{re} \frac{d\delta}{dr} - v_{ye} \right) \tag{35}$$

The term on the right hand side of Eq. (35) is Head's entrainment function, F, and will be evaluated later. Since the density is assumed to be a function of radius only, it comes out of the integral and the integrals can be evaluated for the power law profiles of Eq. (1) which results in the following equation

$$\frac{1}{\delta} \frac{d\delta}{dr} + \frac{1}{v_{re}} \frac{dv_{re}}{dr} + \frac{1}{\rho} \frac{d\rho}{dr} + \frac{1}{r} - \frac{1}{1 + m} \frac{dm}{dr} = \frac{F(1 + m)}{\cos \alpha_e} \tag{36}$$

This is the form desired except for the density term. The density variation can be expressed in terms of the pressure variation for the isentropic core flow as follows

$$\frac{d\rho}{dr} = \frac{1}{a^2} \frac{dp}{dr} \tag{37}$$

Combining this with Eq. (9) then yields

$$\frac{1}{\rho} \frac{d\rho}{dr} = - \frac{M_{re}^2}{v_{re}} \frac{dv_{re}}{dr} + \frac{M_{\theta e}^2}{r} \tag{38}$$

The final form of the continuity equation then becomes

$$\frac{1}{\delta} \frac{d\delta}{dr} + \frac{(1 - M_{re}^2)}{v_{re}} \frac{dv_{re}}{dr} + \frac{1 + M_{\theta e}^2}{r} - \frac{1}{1 + m} \frac{dm}{dr} = \frac{1 + m}{\delta} \frac{F}{\cos \alpha_e} \tag{39}$$

Using similar techniques, the radial and tangential momentum equations can also be reduced to ordinary differential equations.

The entrainment function, F, is given by Eq. (14). The shape factor, H_1, is defined by the expression

$$H_1 = \frac{\delta - \delta_{1S}}{\delta_{2S}} \tag{40}$$

De Ruyck and Hirsch (11) approximated this using the radial velocity components in the definitions of δ_1 and δ_2, thus

$$H_1 = \frac{\delta - \delta_{1r}}{\delta_{2r}} \tag{41}$$

Although Deruyck and Hirsch stated that Eq. (41) is good only for small angles, α, Davis (6) shows that the radial thicknesses, δ_{1r} and δ_{2r}, are the proper thicknesses to use in the definition of H_1 for use in the entrainment function. Using the power law profiles of Eq. (1) in the definition of δ_{1r} and δ_{2r}, the shape factor, H_1, then becomes

$$H_1 = \frac{1 + 2m}{m} \tag{42}$$

and finally the entrainment function, F, becomes

$$F = 0.0306 \left(\frac{m}{1 - m} \right)^{0.653} \tag{43}$$

TABLE I. - FIVE TEST CASES FOR COMPARISON OF ANALYSIS TO RESULTS FROM THE ADD CODE

[Parallel wall diffusers - r_2 = 8.049 cm, r_3/r_2 = 1.1605, h/r_2 = 0.03329, T'/T_{STD} = 1.]

Inlet conditions							
Case	δ/h	m	n	p/p_{STD}	v_{re}/v_{cr}	$v_{\theta c}/v_{cr}$	α_e
I	0.2400	0.2	0.2	0.7087	0.7498	0	0
II	.2400	.2	.2	.9172	.2705	0.2705	45
III	.9556	.2	.2	.9172	.2705	.2705	45
IV	.2400	.2	.2	.7009	.2603	.7151	70
V	.9556	.2	.2	.7009	.2603	.7151	70

TABLE II. - TWO TEST CASES FOR COMPARISON WITH DATA FROM REFERENCE 4

[Parallel wall diffusers - r_2 = 17.64 cm, r_3/r_2 = 1.377, h/r_2 = 0.0605, T'/T_{STD} = 1.]

Inlet conditions							
Case	δ/h	m	n	p/p_{STD}	v_{re}, m/sec	$v_{\theta e}$, m/sec	α_e
VI	0.288	0.2	0.1	1	6.83	7.19	46.5
VII	.360	.5	.2	1	6.35	15.24	67.4

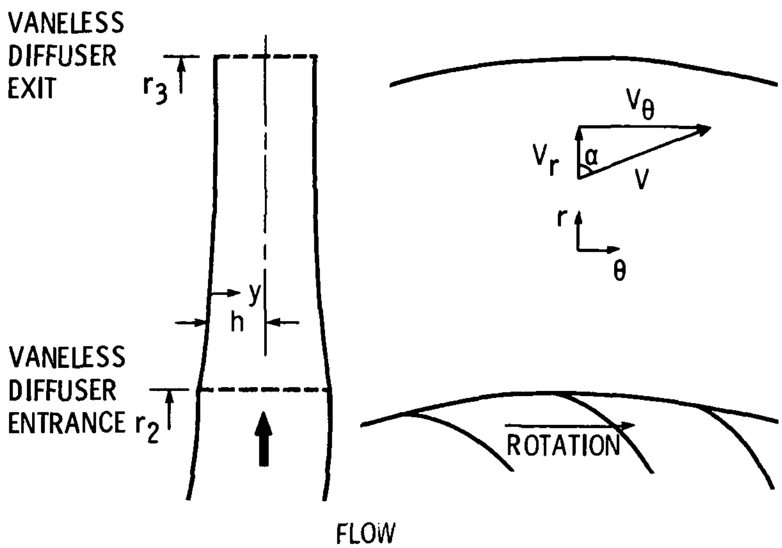
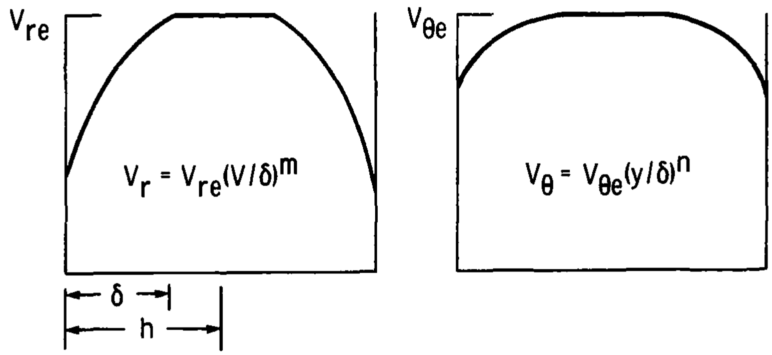
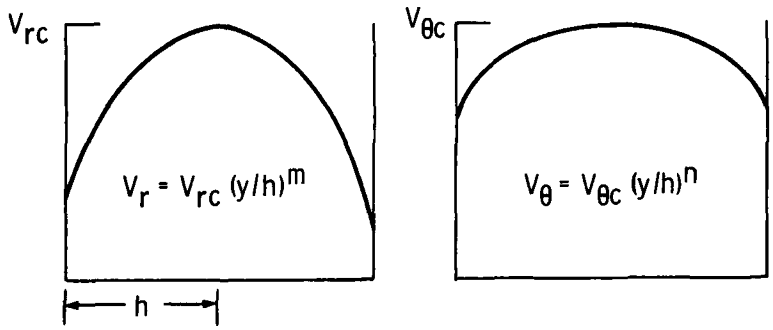


Figure 1. - Vaneless diffuser geometry.

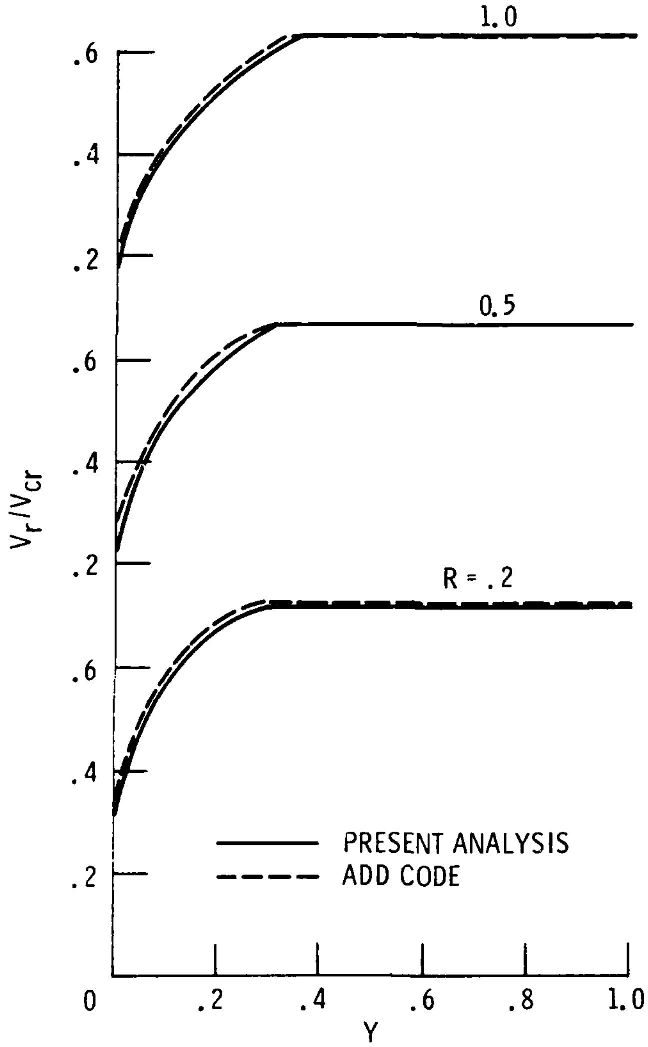


(a) Boundary layer flow.



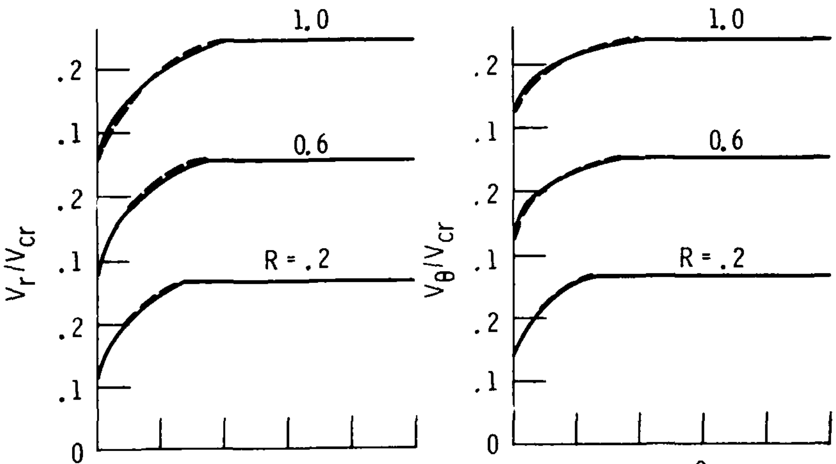
(b) Fully developed flow.

Figure 2. - Typical velocity profiles.

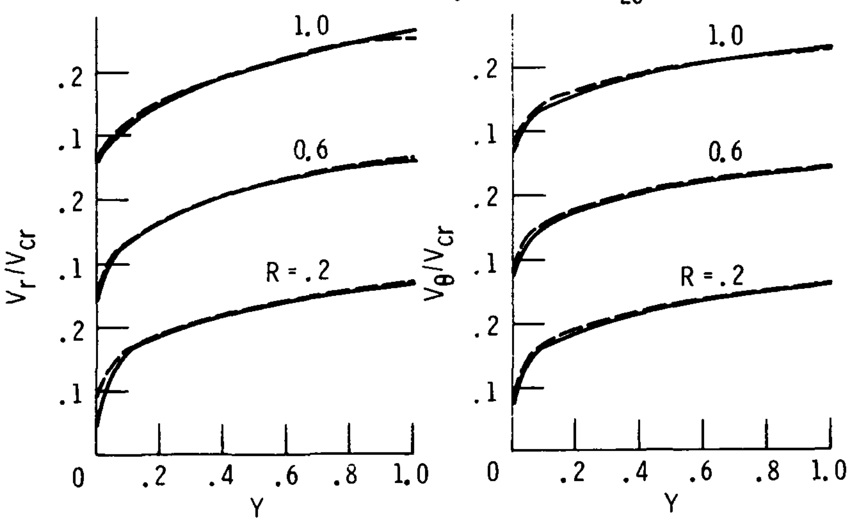


(a) Case I: Boundary layer flow; $\alpha_{2e} = 0^0$.

Figure 3. - Velocity profile comparisons with results from ADD code.

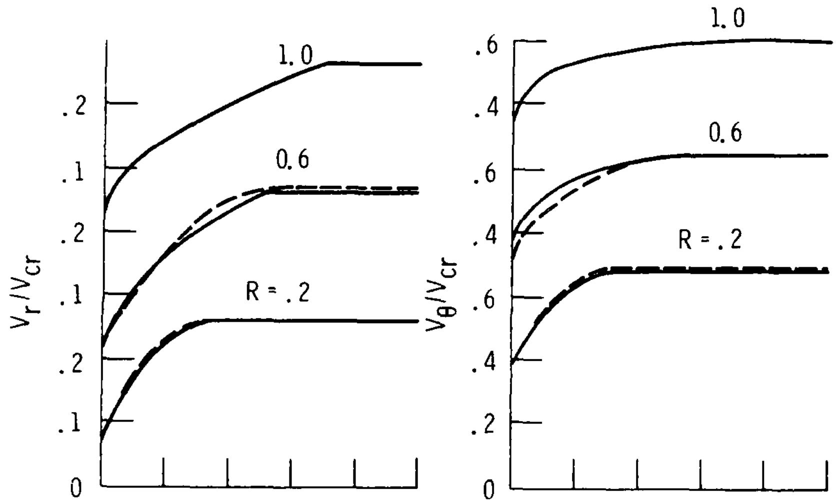


(b) Case II: Boundary layer flow; $\alpha_{2e} = 45^0$.

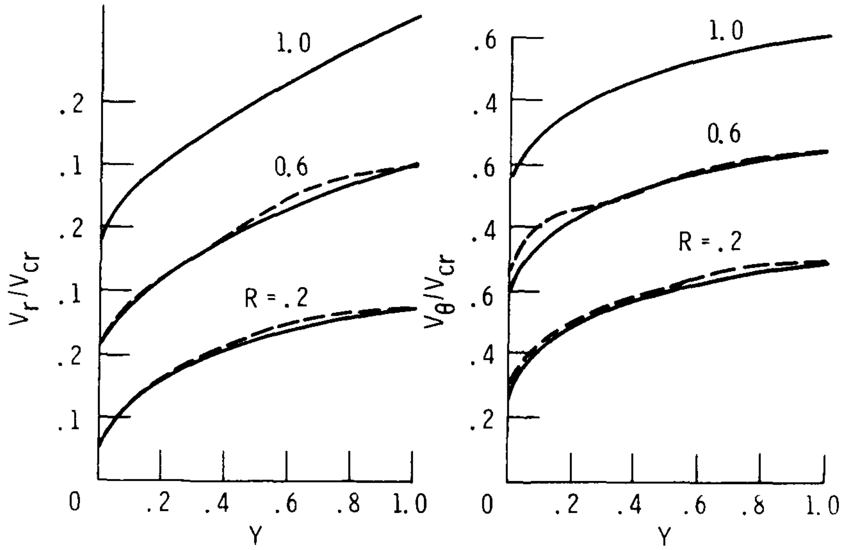


(c) Case III: Fully developed flow; $\alpha_{2e} = 45^0$.

Figure 3. - Continued.



(d) Case IV: Boundary layer flow; $\alpha_{2e} = 70^0$.



(e) Case V: Fully developed flow; $\alpha_{2e} = 70^0$.

Figure 3. - Concluded.

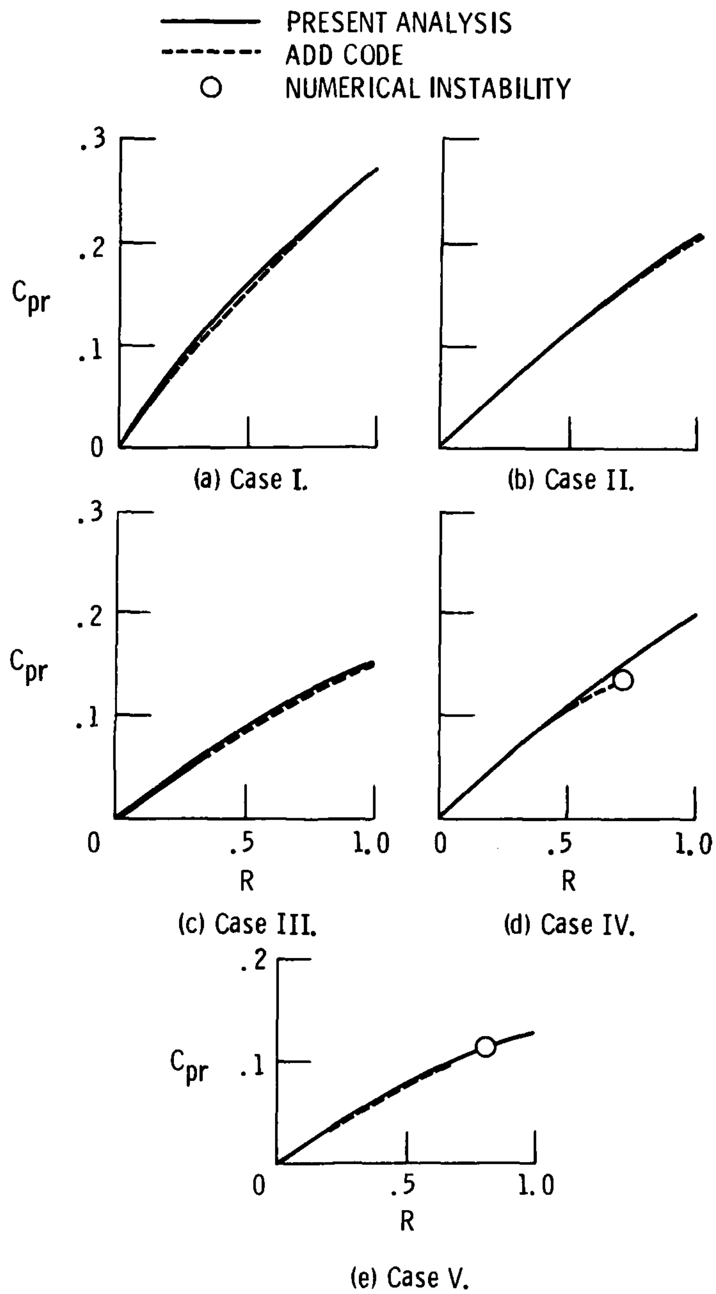


Figure 4. - Static pressure coefficient comparison with ADD code.

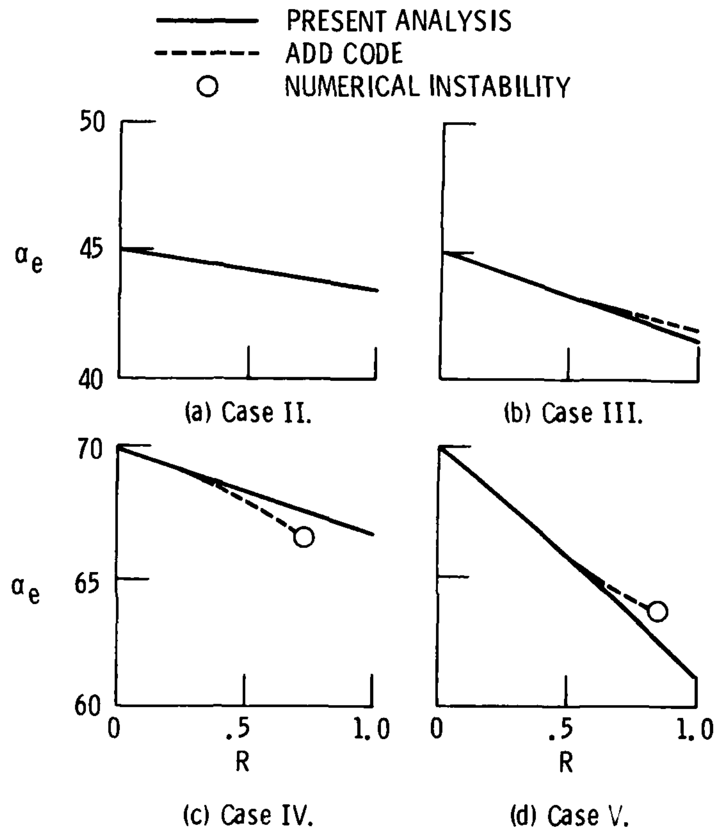
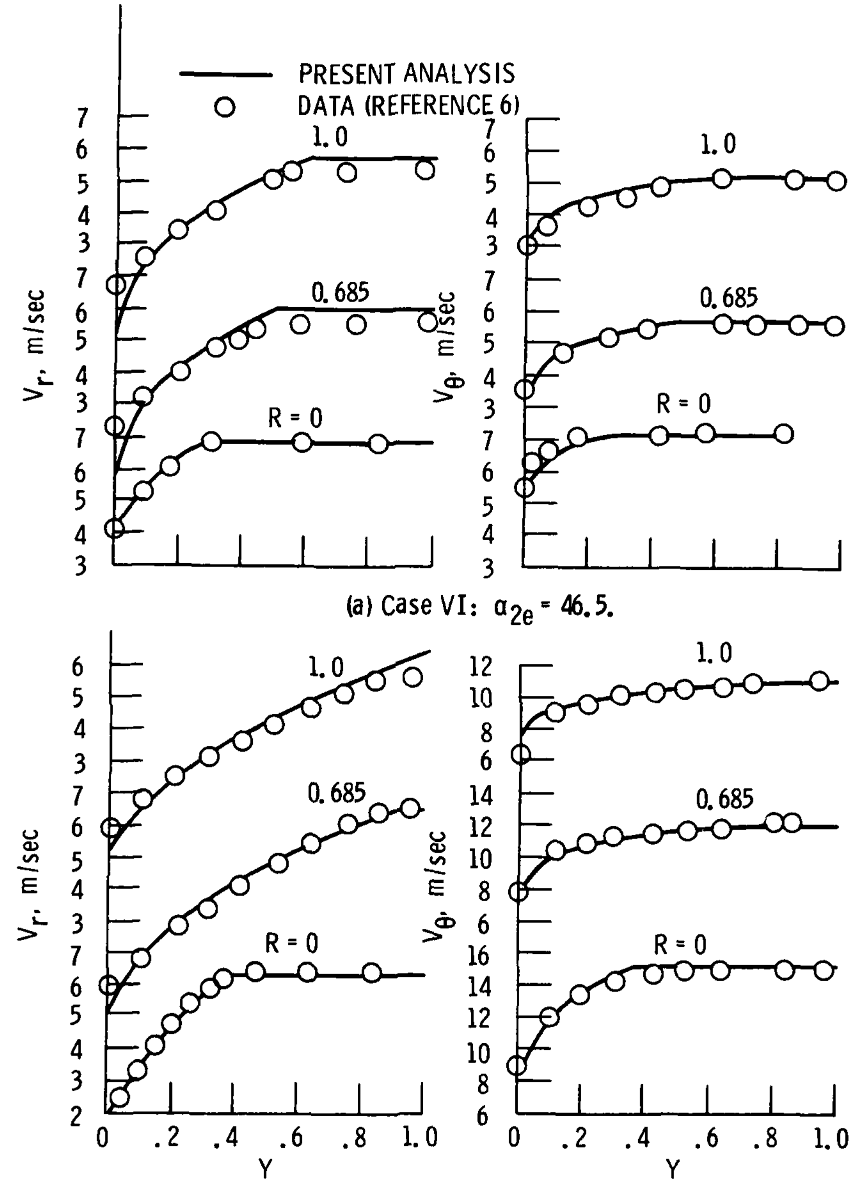


Figure 5. - Centerline flow angle comparison with ADD code.



(a) Case VI: α_{2e} = 46.5.

(b) Case VII: α_{2e} = 67.4.

Figure 6. - Velocity profile comparisons with data from reference 4.

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16. Abstract

An analytical model is proposed to calculate the three-dimensional axisymmetric turbulent flowfield in a radial vaneless diffuser. The model assumes that the radial and tangential boundary layer profiles can be approximated by power law profiles. Then, using the integrated radial and tangential momentum and continuity equations for the boundary layer and corresponding inviscid equations for the core flow, there results six ordinary differential equation in six unknowns which can easily be solved using a Runge-Kutta technique. A model is also proposed for fully developed flow. The results using this technique have been compared with the results from a three-dimensional viscous, axisymmetric duct code and witn experimental data and good quantitative agreement was obtained.

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